the PZTs, the surface area covered by the PZTs, plate dimensions, and the boundary conditions at the edges of the plate. This linear relationship between the intensity of the uniformly distributed load and the requisite voltage is to be expected since the deflection of a point is directly proportional to the load intensity and magnitude of surface forces is proportional to the voltage applied to the PZTs.

We now study the deformations of a cantilever plate with the objective of controlling the deflection of point P located at the center of the free end of the plate; see Fig. 5. It is clear from the results depicted in Fig. 5 that by applying proper voltage to the PZTs the deflection of point P can be made zero. For the cantilever aluminum plate, the voltage to be applied to the actuators to nullify the deflection of point P equaled 439.5 V/mm. We note that the deflection of other points near the fixed end is increased but that of those near the free end is decreased when the PZTs are activated. Keeping the sizes and relative locations of the three PZTs fixed, they were moved horizontally along the x_1 axis. The variation of the electric field intensity with the location on x_1 axis of the PZTs required to make the deflection of point P zero when a point load of 0.1 N is applied at P is shown in Fig. 6. It is apparent that the voltage to be applied to the PZTs increases sharply as the PZTs are moved towards the fixed end of the plate.

Acknowledgments

This work was supported by the U.S. Army Research Office Grant DAAH 04-93-G-0214 to the University of Missouri–Rolla and a matching grant from the Missouri Research and Training Center; the Virginia Polytechnic Institute and State University acted as a subcontractor.

References

[1]Baz, A., and Poh, S., "Performance of an Active Control System with Piezoelectric Actuators," *Journal of Sound and Vibration*, Vol. 126, No. 2, 1988, pp. 327–343.

[2]Halpin, J. C., *Primer on Composite Materials: Analysis*, Technomic, Westport, CT, 1984.

[3]Ghosh, K., "Shape Control of Plates Using Piezoceramic Elements," M.S. Thesis, Univ. of Missouri–Rolla, MO, Aug. 1994.

[4]Crawley, E. F., and de Luis, J., "Use of Piezoelectric Actuators as Elements of Intelligent Structures," *AIAA Journal*, Vol. 25, 1987, pp. 1373–1385.

Thermal Stresses in Eccentrically Stiffened Composite Plates

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Introduction

FIBER reinforced composite materials have found wide applications in aerospace structures. Because these structures are often subjected to thermal loadings, their thermal stress analysis is quite important. The thermal stresses in bare composite plates have been well studied.[1] Recently, the thermally induced geometrically nonlinear response of symmetrically laminated composite plates was investigated by Meyers and Hyer.[2] The formulation of stiffness, thermal expansion, and thermal bending for stiffened composite panels was presented by Collier.[3]

The objective of this Note is to present a method for predicting thermally induced deformations and stresses in eccentrically stiffened composite laminates. The constitutive relations of a laminated composite stiffener are derived. Thermal effect and transverse shear deformation are included in the formulation. Numerical results are obtained for an eccentrically stiffened composite laminate subjected to a temperature change that is uniform within the plane of the plate but has a linear gradient through the thickness. The differences of thermal behaviors between stiffened and bare composite laminates are discussed.

Formulation

By using the first-order shear deformation theory, the laminate constitutive relations can be written in the form

$$\begin{Bmatrix} N \\ M \\ Q \end{Bmatrix} = \begin{bmatrix} A & B & 0 \\ B & D & 0 \\ 0 & 0 & A_s \end{bmatrix} \begin{Bmatrix} \varepsilon \\ \kappa \\ \gamma \end{Bmatrix} - \begin{Bmatrix} N^T \\ M^T \\ 0 \end{Bmatrix} \tag{1}$$

where $\{N\}^t=[N_x\ N_y\ N_{xy}]$, $\{M\}^t=[M_x\ M_y\ M_{xy}]$, and $\{Q\}^t=[Q_y\ Q_x]$ are vectors of the in-plane forces, moments, and transverse shear forces, respectively. Here, $\{N^T\}$ and $\{M^T\}$ are vectors of thermal forces and moments, respectively. The relations between the laminate stiffnesses $[A]$, $[B]$, $[D]$, $[A_s]$ and the ply engineering properties are given in Ref. 4.

When transverse shear deformation is considered, the constitutive relations of a thin, flat, bladelike laminated composite stiffener can be expressed as follows:

$$\begin{Bmatrix} N^s \\ M_x^s \\ M_{xy}^s \\ Q^s \end{Bmatrix} = \begin{bmatrix} A^s & B_1^s & B_6^s & 0 \\ B_1^s & D_{11}^s & D_{16}^s & 0 \\ B_6^s & D_{16}^s & D_{66}^s & 0 \\ 0 & 0 & 0 & A_s^s \end{bmatrix} \begin{Bmatrix} \varepsilon^s \\ \kappa_x^s \\ \kappa_{xy}^s \\ \gamma^s \end{Bmatrix} - \begin{Bmatrix} N^{sT} \\ M_x^{sT} \\ M_{xy}^{sT} \\ 0 \end{Bmatrix} \tag{2}$$

where N^s, M_x^s, M_{xy}^s, and Q^s are the axial force, bending moment, twisting moment, and transverse shear force, respectively. The stiffener stiffnesses are defined as

$$\left(A^s, A_s^s\right) = \int_{-h^s/2}^{h^s/2} \left(Q_{11}^s, 5/6 Q_{55}^s\right) b\, dz$$

$$\left(B_1^s, B_6^s\right) = \int_{-h^s/2}^{h^s/2} \left(Q_{11}^s, Q_{16}^s\right) bz\, dz \tag{3a}$$

$$\left(D_{11}^s, D_{16}^s, D_{66}^s\right) = \int_{-h^s/2}^{h^s/2} \left(Q_{11}^s, Q_{16}^s, Q_{66}^s\right) bz^2\, dz \tag{3b}$$

where h^s and b are the depth and width of the stiffener, respectively. The thermal axial force and bending and twisting moments are given by

$$N^T = \int_{-h^s/2}^{h^s/2} \begin{bmatrix} Q_{11}^s & Q_{16}^s \end{bmatrix} \begin{Bmatrix} \alpha_x \\ \alpha_{xy} \end{Bmatrix} \Delta T\, b\, dz$$

$$\begin{Bmatrix} M_x^T \\ M_{xy}^T \end{Bmatrix} = \int_{-h^s/2}^{h^s/2} \begin{vmatrix} Q_{11}^s & Q_{16}^s \\ Q_{16}^s & Q_{66}^s \end{vmatrix} \begin{Bmatrix} \alpha_x \\ \alpha_{xy} \end{Bmatrix} \Delta T\, bz\, dz \tag{4}$$

where

$$Q_{11}^s = \bar{Q}_{11} - \bar{Q}_{12}^2/\bar{Q}_{22}, \qquad Q_{16}^s = \bar{Q}_{16} - \bar{Q}_{12}\bar{Q}_{26}/\bar{Q}_{22}$$

$$Q_{66}^s = \bar{Q}_{66} - \bar{Q}_{26}^2/\bar{Q}_{22}, \qquad Q_{55}^s = \bar{Q}_{55} - \bar{Q}_{45}^2/\bar{Q}_{44} \tag{5}$$

Here, ΔT is the temperature change from the reference temperature. The expressions for the ply stiffnesses $\bar{Q}_{ij}$ $(i, j = 1, 2, 4, 5, 6)$ and coefficients of thermal expansion α_x, α_{xy} in terms of the ply engineering properties are given in Ref. 4.

Received Feb. 5, 1994; revision received Aug. 10, 1994; accepted for publication Sept. 2, 1994.

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The strain displacement relations of the plate and the stiffener are stated in Ref. 5. The eight-node isoparametric plate element and the three-node isoparametric beam element are employed in this study. Their finite element formulations can also be found in Ref. 5.

Results and Discussion

The formulation and accuracy of the present analysis have been verified by numerical examples. Due to the limitation of length, only thermally induced deformations and stresses in centrally eccentrically stiffened square composite laminates are presented. The stiffened plate is composed of one $(0/\mp45/90)_s$ laminate and one (0/90 deg)$_s$ stiffener. The stiffened laminate is subjected to a linear thermal gradient through the thickness. The temperature rises over the top and the bottom surfaces of the laminate are 300 and 0°C, respectively. The material properties for the plate and stiffener are[4] $E_1 = 130$ GPa, $E_2 = 9.5$ GPa, $G_{12} = G_{13} = 6.0$ GPa, $G_{23} = 0.5G_{12}$, $\mu_{12} = 0.3$, $\alpha_1 = -0.3 \times 10^{-6}$/°C, and $\alpha_2 = 28.1 \times 10^{-6}$/°C. The side length and thickness of the plate are 400 and 4 mm, respectively. The depth and width of the stiffener are 8 and 5 mm, respectively. Due to symmetry, only a quarter plate is analyzed with 4 × 4 mesh.

Figures 1 and 2 show the distributions of the deflection and moment M_x for simply supported (S_1) stiffened and bare laminates under the thermal loading considered, respectively. It can be seen from Fig. 1 that the thermal deformation of the stiffened laminate is different from that of the bare laminate. It is the same as expected that the deflection of the stiffened plate is smaller than that of the bare plate. Besides, it should be noted that the peak values of the deflection occur at different positions in these two cases. The curves in Fig. 2 show that the distribution of the moment M_x of the stiffened laminate is unusual in the vicinity of the stiffened position. Because the stiffener can restrain the rotations of the plate to some extent, the maximum value of the moment M_x of the stiffened laminate occurs at the simply supported boundary instead of the center of the plate.

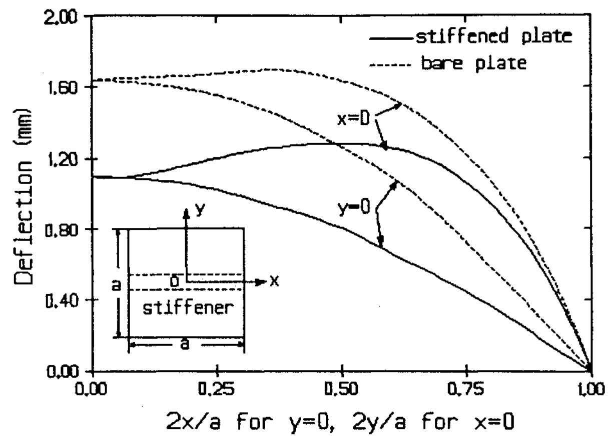

Fig. 1 Deflection of simply supported stiffened and bare square laminates.

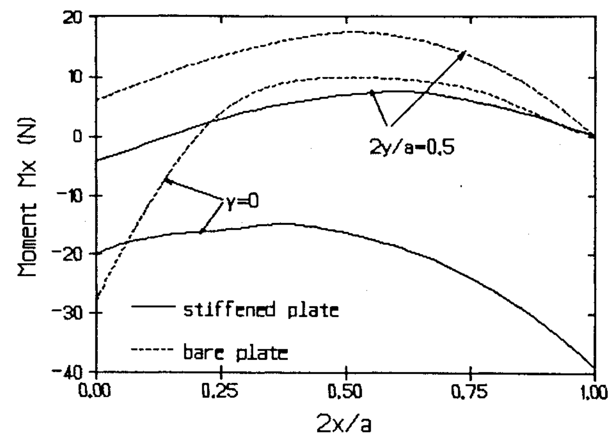

Fig. 2 Moment M_x of simply supported stiffened and bare square laminates.

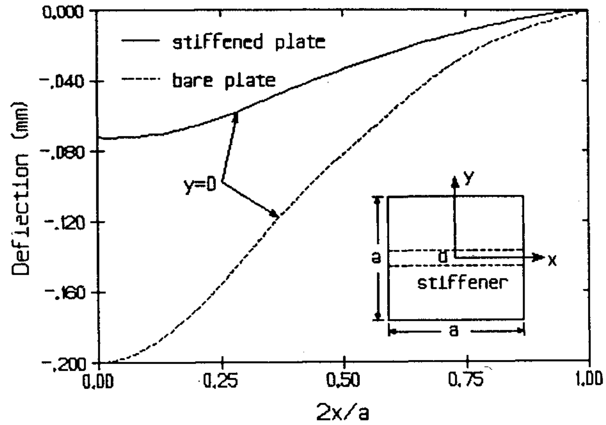

Fig. 3 Deflection of clamped stiffened and bare square laminates.

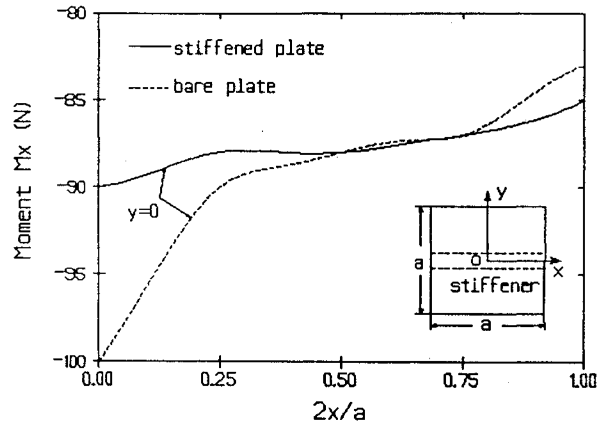

Fig. 4 Moment M_x of clamped stiffened and bare square laminates.

Nevertheless, the distribution of the moment M_x of the stiffened laminate is similar to that of the bare laminate outside the stiffened zone.

Figures 3 and 4 illustrate the distributions of the deflection and moment M_x for clamped stiffened and bare laminates under the thermal loading considered, respectively. Clearly, the distributions of the deflection and moment M_x of the stiffened laminate are similar to those of the bare laminate, but the maximum values of the deflection and the moment M_x of the stiffened laminate are much smaller than those of the bare laminate. This shows that it is suitable to adopt clamped eccentrically stiffened laminates in engineering structures subjected to the thermal loading considered.

Acknowledgment

This investigation is supported by the Research Committee of Hong Kong Polytechnic. The authors are greatly indebted to this institution.

References

[1]Noor, A. K., and Burton, W. S., "Computational Model for High Temperature Multilayered Composite Plates and Shells," *Applied Mechanics Review*, Vol. 45, No. 10, 1992, pp. 419–466.

[2]Meyers, C. A., and Hyer, M. W., "Thermally-Induced Geometrically Nonlinear Response of Symmetrically Laminated Composite Plates," *Composites Engineering*, Vol. 2, No. 1, 1992, pp. 3–20.

[3]Collier, C. S., "Stiffness, Thermal Expansion and Thermal Bending Formulation of Stiffened Fiber-Reinforced Composite Panels," *Collection of Technical Papers—AIAA/ASME Structures, Structural Dynamics and Materials Conference*, AIAA, Washington, DC, 1993, pp. 2279–2290.

[4]Tsai, S. W., and Hahn, H. T., *Introduction to Composite Materials*, Technomic, Westport, CT, 1980.

[5]Cook, R. D., *Concepts and Applications of Finite Element Analysis*, 3rd ed., Wiley, New York, 1981.